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MODELING OF THERMAL PROCESSES IN THERMAL RESISTORS WITH A POSITIVE TEMPERATURE COEFFICIENT OF RESISTANCE

V. N. Shut, S. P. Syrtsov,
E. L. Gavrilenko, and A. V. Gavrilov

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The space-time temperature distribution in materials with a positive temperature coefficient of resistance under the action of electric current on them has been analyzed by numerical methods. The nonstationary boundary-value heat-conduction problem with a nonlinearity of the third kind has been solved. The results obtained theoretically have been compared to experimental data.

Thermal resistors based on a semiconductor $BaTiO_3$ (posistors) have widely been used in electrical engineering and electronics as heating elements, temperature-sensitive elements, elements of protection against current overloads, and others [1]. Such a wide use of them is determined primarily by the presence of a large positive temperature coefficient of resistance (PTCR) observed in the region of temperature of ferroelectric phase transition (T_C) [2]. One can change T_C (and accordingly the temperature of transition to a high-resistance state) and the values of the specific resistance of the material and the PTCR using different modifying additions. Thus, it is possible to create $BaTiO_3$-based thermal resistors with parameters optimized in accordance with the technical conditions of operation of the elements. In certain cases, the use of thermal resistors in circuits of protection against high current overloads and under the conditions of intense heat exchange with the environment is limited by the high thermal stresses, which may give rise to cracks and their propagation or may end in mechanical failure of the samples. Therefore, the problem of calculation of the distribution of temperature in PTCR thermistors and its time variations in the process of switching of the elements to a high-resistance state is of great importance, and its solution has been the objective of a number of investigations [3–8]. It was shown that the temperature difference between the central part of the thermistor and its surface may attain 20–40^{o}C. The temperature gradient substantially depends on the properties of the material (specific resistance, PTCR, and phase-transition temperature) and the conditions of heat exchange with the environment. However, study of the above factors has received little attention.

The dependence of the temperature distribution in samples on the conditions of heat exchange with the environment and the strongly nonlinear relationship between the bulk heat-flux density and the temperature make it necessary to solve the nonstationary boundary-value problem with a nonlinearity of the third kind even in the simplest one-dimensional case [9]. The use of analytical methods of solution of the problem posed shows little promise, and it is expedient to use numerical methods to solve it. This investigation seeks to develop a mathematical model of description of the thermophysical processes in PTCR thermistors.

Theoretical Model. The geometry of posistor elements commonly used in practice and the structure of connecting elements are such that the heat flux may be assumed to propagate in one direction in the system. This enables us to substantially simplify the theoretical description, restricting ourselves to solution of a one-dimensional heat-conduction equation.

We consider a posistor element in the form of a thin plate of thickness L and area S. The temperature distribution over the thickness $T(x)$ $(0 < x < L)$ may be obtained in this case by solution of the one-dimensional heat-conduction equation

$$\lambda \frac{\partial^2 T}{\partial x^2} - c\rho^* \frac{\partial T}{\partial t} + q = 0 , \tag{1}$$

Institute of Engineering Acoustics, National Academy of Sciences of Belarus, 13 Lyudnikov Ave., Vitebsk, 210717, Belarus; email: shut@vitebsk.by. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 78, No. 4, pp. 97–101, July–August, 2005. Original article submitted January 27, 2004; revision submitted March 25, 2004.

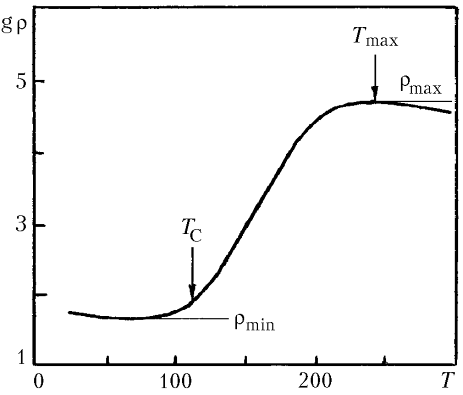


Fig. 1. Specific resistance ρ of a $BaTiO_3$-based posistor ceramics (T_C = 120°C) vs. temperature T. ρ, Ω·cm; T, °C.

satisfying the initial condition $T(x) = T_0 = 20^\circ C$ at $t = 0$.

The boundary conditions have been selected under the assumption that the heat flux on the surface is in proportion to the difference of the surface temperature and the environment temperature (Newton's law):

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} + \sigma\,[T(x, t) - T_0]/\lambda = 0\,, \tag{2}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=L} - \sigma\,[T(x, t) - T_0]/\lambda = 0\,. \tag{3}$$

In Eq. (1), the bulk heat-flux density depends on the temperature of the volume element $q = q(x, T)$. Thus, we have a nonstationary, nonlinear boundary-value problem of the third kind [9]. In the case where the specific resistance $\rho(x, T)$ is temperature- and coordinate-dependent, the relation

$$q(x, T) = \frac{U^2 \rho(x, T)}{\left(\int_0^L \rho(x, T)\, dx\right)^2} \tag{4}$$

holds.

The specific resistance of a posistor ceramics ρ depends on the composition of the material, the technological regimes of its preparation, the sample temperature, and the voltage applied (varistor effect). A typical temperature dependence of the specific resistance of a barium-titanate-based semiconductor ceramics is shown in Fig. 1.

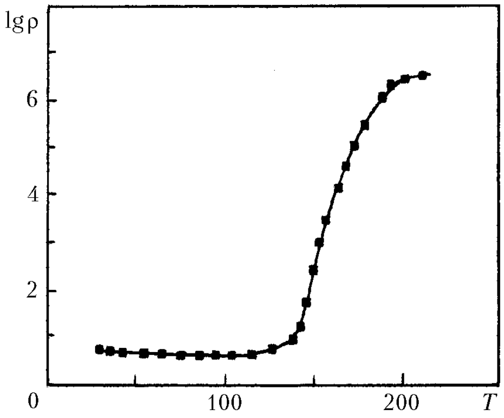
It is difficult to approximate the temperature dependence of the specific resistance throughout the interval of operating temperatures by a single mathematical expression. We used a generalized, or "typed", temperature characteristic that is distinguished for its simplicity and provides a correct idea of the character of variation in the resistance in the materials under study:

$$\rho(x, T) = \begin{cases} \rho_0(x)\,, & T < T_C\,; \\ \rho_0(x) \exp\left\{\alpha\,(T(x) - T_C)\right\}, & T_C \le T < T_{max}\,; \\ \rho_0(x) \exp\left\{\alpha\,(T_{max} - T_C)\right\} & T > T_{max}\,. \end{cases} \tag{5}$$

It is common knowledge that the PTCR effect in ceramic samples is determined by the change in the height of the potential barrier at the grain boundaries in the region of ferroelectric phase transition. It is precisely in this re-

TABLE 1. Characteristics of Posistor-Ceramics Samples

Sample No.	ρ_0, Ω·m	α, %/K	T_C, °C	L, mm	S, mm^2
1	5.1	0.2	130	2.64	285
2	240	0.1	212	2.21	285
3	320	0.1	207	3.50	284



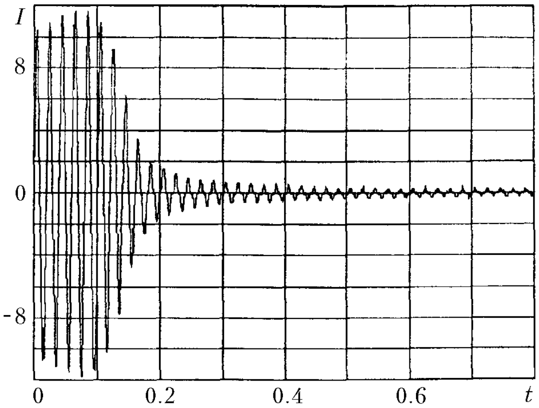


Fig. 2. Temperature dependence of the specific resistance ρ for sample No. 1. ρ, Ω·m; T, °C.

Fig. 3. Oscillogram of the current I for sample No. 1. I, A; t, sec.

gion that the specific heat of a sample undergoes a substantial change. In the calculations, we used the following dependence of the specific heat on the temperature [3]:

$$c(\theta) = c_0 [1 + c_1\theta + c_2 \exp(\pm \lambda\theta)] , \tag{6}$$

where c_0 = 523 J/K, c_1 = 0.024 J/K, c_2 = 0.296 J/K, and $\lambda = \pm 3$ (+ for $\theta < 0$ and − for $\theta > 0$). The dimensionless temperature θ is determined from the equation

$$\theta = (T - T_C)/T_C . \tag{7}$$

The remaining characteristics of the elements (λ, σ, and others) do not undergo such a sharp change in the region of temperatures under study, and we disregarded their temperature dependence in the calculations.

The problem was solved by the finite-difference method according to the six-point scheme (Crank–Nicholson scheme). The algorithm was realized with the use of object-oriented programming in the Delphi environment. We have developed a convenient interface for the MS Windows 2000 operating system to change the thermistor parameters and to analyze the results obtained.

Check of the Model. The correctness of the mathematical model proposed and of the algorithm of its numerical solution was checked by comparison of theoretical results to experimental data. For this purpose, we used the samples of thermistors with known characteristics (temperature dependence of the resistance and surface temperature of the samples and their geometric dimensions). The surface temperature was measured in accordance with the recommendations of the International Electrotechnical Commission. The voltage applied to the samples was 220 V. The characteristics of the samples (specific resistance at room temperature, positive temperature coefficient, phase-transition temperature, thickness, and area) are given in Table 1.

The temperature dependence of the specific resistance for sample No. 1 is given in Fig. 2.

The switching time was determined as the period of the thermistor attaining the temperature at which its resistance is twice as high as the minimum resistance (based on the dependence $R = f(T)$). To experimentally determine this parameter we investigated the dependences of the current traversing the sample on the time. It is clear that the

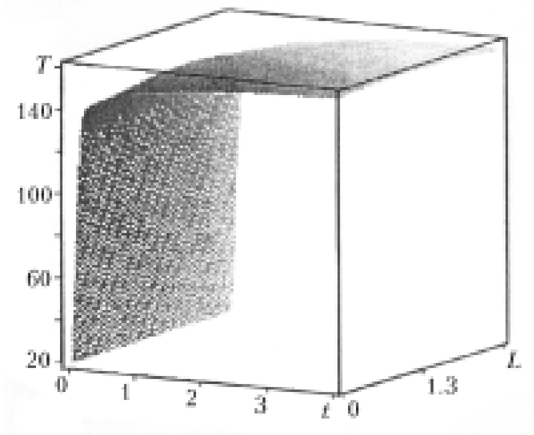


Fig. 4. Variation in the temperature T over the thickness L of sample No. 1 with time. T, °C; t, sec; L, mm.

TABLE 2. Calculated and Experimental Values of the Surface Temperature and Switching Time of Posistor-Ceramics Samples

Sample No.	T_s, °C		t_r, °C	
	experiment	calculation	experiment	calculation
1	157	157.8	0.16	0.17
2	229	239	3.2	3.6
3	215	225.5	15	17

switching time corresponds to the period when the current becomes half as high as its maximum value. The oscillogram of the current for sample No. 1 is given in Fig. 3.

Based on the oscillograms of the current, we made corrections to the value of the specific resistance. The resistances of the samples in heating by the electric current are lower than the resistances measured at the zero power because of the varistor effect. Furthermore, posistor ceramics is characterized by the presence of a negative coefficient of resistance at temperatures lower than the temperature of ferroelectric phase transition. This phenomenon was also taken into account in the calculations by correction of the value of ρ_0. The variation in the temperature with time for sample No. 1 is given in Fig. 4.

Analogous measurements were also carried out for other samples. The calculated and experimental values of the surface temperature and the switching time are systematized in Table 2, which shows a good agreement between experimental and theoretical results.

Thus, the model proposed may be used for analysis of thermophysical processes in materials with a PTCR.

NOTATION

c, specific heat, J/(kg·K); I, current, A; L, thickness of the samples, m; q, bulk density of the heat flux, W/m^3; R, resistance of the posistor, Ω; S, area of the samples, m^2; T, temperature, °C; T_C, Curie temperature, °C; T_0, initial temperature (environment temperature), °C; T_{max} and T_{min}, maximum and minimum temperatures of the posistors, °C; T_s, surface temperature of the thermistors, °C; t_r, switching tome of the thermistor, sec; U, voltage applied to the posistor V; α, positive temperature coefficient of resistance, %/K; θ, dimensionless temperature; λ, thermal conductivity, W/(m·K); ρ^*, density, kg/m^3; ρ, specific resistance, Ω·m; ρ_0, specific resistance at 20°C, Ω·m; σ, coefficient of heat transfer of the surface, W/(m^2·K). Subscripts: max, maximum; min, minimum; s, surface; r, refraction (switching); 0, initial value.

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